Vertical Integration and Innovation*

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Abstract

We studied the effect of vertical integration on investment incentives and social welfare when both upstream and downstream firms make innovative investments. When only upstream investment is needed for the final product, vertical integration improves coordination inside the integrated entity, which increases the investment incentive of the integrated upstream firm and decreases that of the independent upstream firm. The integrated upstream firm has excessive investment incentive and social welfare is lower under vertical integration if product differentiation is limited. When only downstream investment is needed, vertical integration has no effect on investment incentives. When both upstream and downstream investments are needed, investment incentives are insufficient. Vertical integration enhances both upstream and downstream investment incentives but also crowds out more investment from the independent firms. The net effect on social welfare is positive.

Key Words: Vertical Integration, Innovation, Complementarity

JEL Classification: D4, L1, L4

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1 Introduction

In a number of vertically related industries, innovative investments take place at both upstream and downstream levels. Take smartphone for an example, upstream market consists of those firms that develop technology; while downstream market includes those who design and manufacture cellphones. Vertical integration is a common practice in this industry. However, there are also independent upstream developers and independent downstream manufacturers. Our purpose in this paper is to study the impact of vertical integration on investment incentives and social welfare when both upstream and downstream firms take active roles in the innovative process.

We present a model with duopoly in both upstream and downstream markets. Our first result concerns the case when only upstream innovation is needed for the industry, i.e. downstream firms do not make investment decisions but they are the necessary channels to final consumers. In this situation, partial integration between one upstream firm and one downstream firm improves coordination inside the integrated entity, which increases the investment incentive of the integrated upstream firm and decreases that of the independent one. This is because in case that both upstream firms make successful investments, they only compete for the independent downstream firm but not for the integrated downstream firm. This leaves the integrated upstream firm positive profit when upstream firms compete a-la-Bertrand to supply downstream firms, but the independent upstream firm only gets zero profit. However, improved investment incentive of the integrated upstream firm may not be socially beneficial, instead it may be excessive. The intuition is simple: given that one upstream firm has already made a successful investment, the social value from the investment of a second upstream firm may be less than the private gain. And therefore, when only upstream innovation matters, vertical integration may lead to overall over-investment and hurts social welfare.

Downstream investment changes this scenario in two ways. First, downstream competition is not always present, and thus the upstream firm cannot catch the whole profit of downstream market even when she is the only upstream innovator, since downstream innovation is also necessary for the final product. Second, when upstream firms and downstream firms bargain after the realization of the outcomes of investments, we encounter the classical hold-up problem. Under vertical integration, the upstream investment crowd-out effect still exists, which leads to higher investment from the integrated upstream firm. However, this improved incentive may not be excessive, instead it may be welfare enhancing. This is because under vertical separation, upstream firms under-invests compared to social optimum since their private gains are lower than that of the social planner. Moreover, vertical integration solves the hold-up problem inside the integrated entity, this has two effects. First, the integrated downstream firm has higher incentive to invest and then crowds out the investment of the independent downstream firm. Second, when there is only one successful upstream firm, the profit for an integrated upstream monopolist is higher than that of a separated one, which further increases the investment incentive for an integrated upstream firm. In this sense, downstream investment amplifies the upstream crowd-out effect of vertical integration. With quadratic cost function, we show that vertical integration actually increases social welfare.
Our paper contributes to the literature on the foreclosure effect of vertical integration, which dates back to the seminal paper of Ordover, Saloner and Salop (1990). Most of the literatures along this line focus on the downstream foreclosure effect, and our result can be interpreted as the upstream foreclosure effect of vertical integration. With a focus on the role of investments, most of previous works have only upstream firms invest or only downstream firms invest. Bolton and Whinston (1993) studied the effect of vertical integration on downstream investments. In their model, whether there is downstream competition ex post exogenously depends on whether there are one or two units of input available. On the contrary, competition is endogenously determined by downstream investment in our model, and vertical integration softens downstream competition.

Chen and Sappington (2010) studied upstream investment incentive with a monopolistic upstream firm, they show that vertical integration generally enhances upstream innovation under downstream Cournot competition but may diminish upstream innovation with downstream Bertrand competition. Since there is no upstream competition, upstream foreclosure does not exist there. Our paper is a first attempt to the problem with both upstream and downstream investments.

Our paper is also related to some literatures discussing the effect of integration on innovation of complementary products. Farrell and Katz (2000) shows that integration into a complementary product market allows a monopolist to extract more rent from the market where he dominates. Schmidt (2009) studies how vertical integration of an patent holder affect the contractual terms between upstream patent holders and downstream producers. In these papers, the complementarity is between horizontally related products. In our paper, such complementarity is vertical.

The paper proceeds as follows: We present the basic framework in Section 2; Section 3 studies two benchmarks, when only upstream innovation or only downstream innovation is needed for the final product; Section 4 presents the results with both upstream and downstream innovation matter for the final product; Section 5 provides some extension and discussion; Section 6 concludes. All proofs are shown in the Appendix.

2 The Framework

Players and Market Interaction The industry consists of an upstream market and a downstream market. There are two upstream firms $U_A$ and $U_B$, and they supply input for two downstream firms $D_1$ and $D_2$. Each $D_j, j = 1, 2$ demands only one unit of the input. We assume that once $D_j$ fails to trade with both $U_i, i = A, B$, he has no alternative source for the input. Similarly, if $U_i$ fails to trade with both downstream firms, she has no other ways to access the final consumer market.

Product Market The payoff to each downstream firm depends on whether there is competition in the final good market. For each downstream firm $D_j$, if he is the only active one in the downstream market, he gets profit $\Delta$; if both downstream firms are active, each $D_j$ can only get profit of $\delta$. We assume that $0 < 2\delta < \Delta$, so that competition dissipates part of the profit but not all of it. Thus the payoff for downstream firms is described in the following table,
Table 1:

<table>
<thead>
<tr>
<th>D1</th>
<th>D2</th>
<th>A</th>
<th>NA</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>δ,δ</td>
<td>Δ,0</td>
<td>NA</td>
</tr>
<tr>
<td>NA</td>
<td>0,Δ</td>
<td>0,0</td>
<td></td>
</tr>
</tbody>
</table>

where “A” and “NA” indicate whether \( D_j \) is active or not active in the downstream market. Therefore, if \( U_i \) is the only upstream innovator, the industry profit is maximized when she only sells to one of the two downstream firms.

Technology A successful final product may need innovative investment from both upstream and downstream firms. Each upstream firm \( U_i \) can make investment in innovation, which in case of success enables her to supply the input; otherwise, \( U_i \) has no successful innovation and has to stay out of the market. We model the upstream innovation in the following way: for a given level of investment \( e_i \), \( U_i \) succeeds in innovation with probability \( e_i \) and fails with the complementary probability \( 1 - e_i \). The cost of investment is the same for both upstream firms, which is given by \( C_U(e) \). We assume that \( C_U(e) \) is increasing and convex in \( e \). We assume that there is no marginal cost of production. Thus, the total cost for an upstream firm is the fixed cost of investment. Furthermore, there is no capacity constraint or any other shocks that may constrain the production of \( U_i \), and each \( U_i \) can supply both downstream firms if she is willing to.

Each downstream firm \( D_j \) also makes investment in innovation, which in case of success allows him to transform input to final product on a one-to-one basis at zero cost. As the upstream market, we model downstream investment as follows: each downstream firm \( D_j \) makes an investment \( d_j \) which succeeds with probability \( d_j \) and fails with probability \( 1 - d_j \). The cost of investment is \( C_D(d) \), which is convex and increasing.

We further make the following assumptions on the cost function of upstream and downstream investments,

**Assumption 1.** \( C'_U(1) > \Delta \); and \( C'_U(e) > 0 \), \( C''_U(e) > \Delta \) for \( e \in [0,1] \).

**Assumption 2.** \( C'_D(1) > \Delta \); and \( C'_D(d) > 0 \), \( C''_D(d) > \Delta \) for \( d \in [0,1] \).

The above assumptions guarantee that the profit functions are well-defined and the solutions are interior. In the special case of quadratic cost function \( C_U(e) = \frac{1}{2}c_u e^2 \) and \( C_D(d) = \frac{1}{2}c_d d^2 \), the above assumptions amount to say \( c_u > \Delta \) and \( c_d > \Delta \).

**Timing of the Game** Throughout the paper, we study the investment game with time-line as follows:

- **Stage 1: Upstream Investment** Upstream firms choose their investment \( e_i, i = A, B \), and the outcomes of upstream investments realize;

- **Stage 2: Downstream Investment** Observing the outcome of upstream investment, downstream firms choose their investment \( d_j, i = 1, 2 \), and the outcomes of downstream investments realize;
- **Stage 3: Bargaining** Successful upstream firms and downstream firms bargain over the terms of inputs supply; and inputs are delivered according to all agreements between firms;

- **Stage 4: Final Product Market**: Final product market and payoff to downstream firms realize.

We make two comments about the time-line. First, we assume that the downstream firms invest after observing the outcomes of upstream investments. This fits a number of situations. For instance, downstream distributors do marketing happens after upstream firms have successfully introduced some new products; downstream developers start to develop final products after upstream innovator have make successful discoveries. Moreover, with such sequential investment, we are able to eliminate some socially wasteful downstream investment. Since if both upstream firms fail in investment, there would be no value of downstream investment. Second, we assume that bargaining happens after the realization of outcomes of both upstream and downstream investments. This allows us to study the interaction between upstream and downstream investments. If bargaining happens in an interim stage, i.e. the successful upstream innovators bargain with downstream firms after the realization of upstream investment but before the investment of downstream firms, then downstream investment incentives won’t be affected by the bargaining process or whether there is vertical integration.

*Bargaining* We propose a simple bargaining procedure between (successful) upstream firms and (successful) downstream firms. We allow the offers to have an exclusive dealing clause. A typical offer would be either \((p)\) or \((p, E)\), where \(p\) is the price of the input, and \(E\) specifies whether exclusive dealing is required. With equal probability, either upstream firms or downstream firms are chosen to make offers. For instance, if upstream firms were chosen, they make offers to downstream firms, downstream firms make acceptance or rejection decisions; inputs are delivered according to accepted offers, and game ends. It is the same if downstream firms are chosen to make offers.

We assume that whenever a firm is indifferent between accepting and rejecting an offer, he chooses to accept. Moreover, we assume that whenever a firm accept an exclusive dealing offer, he has to turn down all the other offers; and all offers and acceptance decisions are publicly observable.\(^1\)

### 3 Two Benchmarks

To distinguish the main forces at work, we start with two benchmarks where only one-side innovation, either upstream or downstream, is needed for the final product. We study the interaction between upstream and downstream innovation in the next section. In this two

\(^1\)Instead of randomly choosing one side to make offers, we can also specify the bargaining process as a modified version of the Rubinstein’s alternative offer Bargaining game: with either side make simultaneous offer to the other side in period 1; and then those firms who hasn’t exit the bargaining make offers in period 2; and so on. This infinite bargaining procedure would give the same payoff as the simple bargaining procedure we have.
benchmarks, the timing of investment does not matter, since investment on one side is not a concern any more. When only upstream innovation is needed, vertical integration improves the coordination between the integrated firms, which increases the investment incentive of the integrated upstream firm and decreases that of the independent upstream firm. Moreover, partial integration lowers social welfare when product differentiation is limited in the downstream market. On the other hand, when only downstream innovation is needed, vertical integration has no effect on downstream investment incentives and social welfare.

3.1 Only Upstream Innovation is Needed

We start with the first benchmark case where only upstream innovation is needed for the final product. In this situation, as long as the upstream investment is successful, the downstream firms can transform the inputs into final products. That is to say downstream firms do not directly contribute to the value of the final product, they specialize in developing channels to final consumers. We begin with the case when all firms remain separated.

3.1.1 Vertical Separation

Since downstream investment is not needed, both downstream firms are viable for the successful upstream innovator. We need to specify the bargaining outcome depending on whether only one upstream firm or both upstream firms have made successful investment.

If only one upstream firm has successful innovation, downstream competition drives the input price up to $\Delta$, and the upstream monopolist gets the whole downstream market profit $\Delta$. When both upstream firms obtain successful innovation, they compete to sell to downstream firms a-la-Bertrand, which drives down the input price to zero. We show in detail in the Appendix that the bargaining process leads to the payoff matrix as in Lemma 1.

**Lemma 1.** The payoff matrix for upstream firms when only upstream innovation is need is given by where “S” and “F” indicate whether $U_i$ succeeds or fails in investment.

<table>
<thead>
<tr>
<th>$U_A$</th>
<th>$U_B$</th>
<th>S</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0,0</td>
<td>$\Delta$,0</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0,$\Delta$</td>
<td>0,0</td>
<td></td>
</tr>
</tbody>
</table>

**Proof.** See Appendix.

Then for each upstream firm $U_i$, the problem is to choose an effort level to maximize expected profit

$$e_i^* = \arg\max_{e_i}\{e_i(1-e_{i'})\Delta - C_{U}(e_i)\}, i' \neq i$$

which gives us the best response of $U_i$, given the investment level of $U_{i'}$

$$e_i^* = C'_{U}^{-1}((1-e_{i'})\Delta), i' \neq i$$
It is easy to see that \( \frac{\partial e_i^*}{\partial e_i'} = -\frac{\Delta}{C_U'(e_i^*)} < 0 \), and thus the investments of the two upstream firms are strategic substitutes. As \( U_i \) increases investment, it is more likely that \( U_i \) cannot recoup benefit of his innovation and then \( U_i \) has less incentive to invest.

Moreover, from Assumption 1, it is clear that \( -1 < \frac{\partial e_i^*}{\partial e_i'} < 0 \) and then there exists a unique equilibrium in the investment game \( e_{VS}^* \), which is given by the solution of the equation

\[
e_{VS}^* \text{solves } \Delta(1 - e) = C_U'(e)
\]

### 3.1.2 Partial Vertical Integration

Suppose now there is partial vertical integration, i.e. only one upstream firm and one downstream firm integrate, without loss of generality, we assume that \( U_A \) and \( D_1 \) now integrate, while \( U_B \) and \( D_2 \) remain separated.

Compared to the case of vertical separation, there are a few differences. First, if the integrated entity \( U_A - D_1 \) was the only successful firm in the upstream market, \( U_A \) is an upstream monopolist and can extract all downstream market profit \( \Delta \). We assume that in this case \( U_A \) would only supply his own downstream affiliate \( D_1 \) and \( D_2 \) is excluded from the market.

Second, when the independent upstream firm \( U_B \) is the only one who has successful upstream innovation, we do not model any information leakage risk, which would give the independent downstream firm \( D_2 \) certain advantage vis-a-vis the integrated downstream firm \( D_1 \). For example, \( U_B \) may face the risk that if she supplies inputs to \( D_1 \), \( D_1 \) may leak some key information to \( U_A \), and then \( U_A \) would also have successful innovation and compete with \( U_B \). This would make \( D_1 \) as an inferior downstream firm in the perspective of \( U_B \), which grants \( D_2 \) a stronger bargaining position. This type of information leakage problem has been studied in a few other papers such as Allain et al(2011) and Chen(2011). Our focus here is not on how vertical integration affects the information flows in the industry, hence we assume away any information problem. Moreover, in our setting of sequential moves, upstream innovation happens before the actual production of final product, and thus such backward information flow won’t be a big concern. Therefore, when \( U_B \) is the sole upstream innovator, she is still able to extract the whole downstream market profit \( \Delta \) due to downstream competition.

Third, when both upstream firms succeed in investment, the integrated downstream firm \( D_1 \) is always supplied by \( U_A \). And then the bargaining happens between \( U_B, D_2 \) and \( U_A - D_1 \) who acts as an upstream supplier. Upstream competition drives the input price for \( D_2 \) down to zero, and thus the profit for \( U_B \) is zero while \( U_A \) can still catch part of downstream market profit, which is \( \delta \) since now both \( D_1 \) and \( D_2 \) are active in the downstream market.

In sum, the payoff matrix for \( U_A - D_1 \) and \( U_B \) is given by

<table>
<thead>
<tr>
<th>( U_A - D_1 ) ( U_B )</th>
<th>S</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>( \delta,0 )</td>
<td>( \Delta,0 )</td>
</tr>
<tr>
<td>F</td>
<td>( 0,\Delta )</td>
<td>( 0,0 )</td>
</tr>
</tbody>
</table>
Then the upstream firms choose \( e_{1I} \) and \( e_{2I} \) such that

\[
e_{1I} = \arg\max_{e_1} \{ e_1(1 - e_2)\Delta + e_1e_2\delta - C_U(e_1) \}
\]

and

\[
e_{2I} = \arg\max_{e_2} \{ e_2(1 - e_1)\Delta - C_U(e_2) \}
\]

Under Assumption 1, it is easy to see that \( \frac{\partial e_{1I}}{\partial e_2} = -\Delta\frac{\Delta-\delta}{C_U'(e_{1I})} \in (-1,0) \), and \( \frac{\partial e_{2I}}{\partial e_1} = -\Delta\frac{\Delta-\delta}{C_U'(e_{2I})} \in (-1,0) \). Hence the upstream investment game under vertical integration has a unique solution which solves the two best response functions,

\[
\begin{cases}
C'_U(e_{1I}) = \Delta(1 - e_{2I}^2) + \delta e_{2I}^2 \\
C'_U(e_{2I}) = \Delta(1 - e_{1I})
\end{cases}
\]  

(1)

Compared to the situation under vertical separation, we have the following proposition,

**Proposition 1.** The integrated upstream firm invests more than the independent upstream firm. Indeed, we have \( e_{1I} > e_{VS} > e_{2I} \).

**Proof.** See Appendix. \( \Box \)

The proposition is shown in Figure 1. The equilibrium investment is determined by the intersection of the two best response curves \( BR^1(e_2) \) and \( BR^2(e_1) \). The best response function for the independent upstream firm \( U_B \) is not affected by integration. However, integration of \( U_A \) and \( D_1 \) leads to a clockwise rotation of the best response function of \( U_A \), which clearly shows that the integrated upstream firm \( U_A \) invests more than under vertical separation while the independent upstream firm \( U_B \) invests less.

![Figure 1: Equilibrium Upstream Investment under Vertical Separation and Vertical Integration](image)

The stronger investment incentive of the integrated upstream firm originates from the better coordination inside the integrated entity \( U_A - D_1 \). Under vertical integration, when both upstream firms make successful investment, they only compete for the independent downstream firm; and thus the integrated upstream firm is still able to earn positive profit rather than zero under vertical separation, which increases her incentive to invest. Since upstream investments are strategic substitutes, the independent upstream firm invests less because there is a larger chance that the integrated firm would have successful innovation.
3.1.3 Welfare Implications

The welfare effect of vertical integration comes from two factors. On the cost side, the total cost of innovative investment increases. Such increase results from two effects: first, the total level of investment is higher under vertical integration, as the Best Response function has a slope less than one; second, the cost function is convex, so that any asymmetry between the investment levels would increase the total cost.

On the benefit side, the total probability that there will be at least one upstream innovation is higher under vertical integration, which is beneficial. Moreover, when both upstream firms succeed, there may be social gain from product differentiation. Under vertical separation, the upstream firm is not able to capture this benefit of product differentiation; under vertical integration, the integrated firm is able to catch some of this benefit, which boosts his investment incentive. However, the private gain of individual firm may exceed the social gain and that may lead to over-investment and lower social welfare.

To see the net effect of vertical integration on social welfare, we need to specify the downstream final product market. We proceed with the simple Hotelling-line model. The two downstream firms are located at the end point of a Hotelling line with length 1. A representative consumer is randomly located on the line according to uniform distribution, and the consumer has valuation \( v \) for the product. In addition, the consumer incurs a transportation cost which is \( t \) per unit distance to each downstream firm. We assume that \( v \) is large enough compared to \( t \), i.e. \( v > 2t \), hence the market is fully covered no matter there is one or two downstream firms.

Basic result of the simple Hotelling model shows that, when there is only one downstream firm, he charges price \( v - t \) and the profit \( \Delta = v - t \); when there are two downstream firms, they charge the same price \( p = t \) and get profit \( \delta = \frac{t}{2} \). From the social planner’s perspective, when there is only one downstream firm, the social surplus from the final product market is \( v - \frac{t}{2} \). When there are two downstream firms, this social surplus becomes \( v - \frac{t}{4} \). And thus, given one firm is already active in the downstream market, the social gain of a second firm is \( \frac{t}{4} \); while the private gain of the second firm is \( \frac{t}{2} \), which is larger than the social gain. We consider the quadratic cost function with \( c_u = v = 1 \), and we have the following result.

**Proposition 2.** Consider the Hotelling model above, when only upstream innovation is needed, if product differentiation is weak (\( t \) is small), social welfare is lower under (partial) vertical integration than under vertical separation; if product differentiation is strong (\( t \) is big), (partial) vertical integration improves social welfare.

*Proof.* See Appendix.

Most of the reasoning behind the welfare implication is conveyed in the above discussion. When the product differentiation in the downstream market is weak, the welfare gain from more upstream investment is weak; and the negative effect of over-investment from the integrated upstream firm dominates. When there is stronger product differentiation in the downstream market, the positive effect of higher upstream investment dominates and vertical integration increases social welfare.
The above proposition is proved with specific assumptions. Generally, the proposition always holds for relatively small investment cost, as the numerical example shows in Figure 2 (where we set $v = 1$). For relatively small investment cost, vertical integration reduces social welfare if product differentiation is weak. For relatively large investment cost, vertical integration increases social welfare. Since upstream investment incentive is weak with large investment cost, this makes it valuable for the upstream firm to integrate and exert more effort to innovate.

**Remark** In a slightly different model, we have an even stronger result. Suppose the locations of downstream firms are not fixed at the end points of the Hotelling line before innovative investment. Instead, the active downstream firms can choose their location after the investment and bargaining stage. This fits the situation where downstream innovations themselves are not differentiated, it is downstream marketing strategies that differentiate the downstream firms. In this modified model, if only one downstream firm is active, he would choose to locate at the mid-point of the Hotelling line, and get profit $\Delta = v - \frac{t}{2}$; if both downstream firms are active in the market, they would differentiate themselves by locating at the end points of the Hotelling line and each gets profit $\delta = \frac{t}{2}$. However, in this modified Hotelling model, the social surplus in the downstream market when there is one downstream firm active is $v - \frac{t}{4}$; and it remains to be the same $v - \frac{t}{4}$ when there are two downstream firms active.

Therefore, in this case, if the social planner can only control the upstream investment but not downstream firms’ marketing strategies, there is no social gain from a second upstream innovation if one upstream firm has already made successful investment. However, the private gain for a second upstream innovation is $\frac{t}{2}$, which generates excessive upstream investment incentives. Notice that in this different model of downstream market, there is still gain from vertical integration: as an upstream monopolist is not able to catch all consumer surplus from the downstream market, the upstream investment incentive is insufficient under vertical separation. Vertical integration can improve the investment incentives. However, the range of parameters that vertical integration increases social welfare shrinks in this modified model, as the excessive investment incentive is stronger.
3.2 Only Downstream Innovation is Needed

Now we turn to the case when only downstream innovation is needed for the final product, i.e. the input is sort of generic product which cannot be used directly by the final consumers. Downstream firms need to make investments which enables them to transform the input into final product in case of success.

Clearly, in this case, the presence of upstream competition drives the input price down to zero. And therefore, each downstream firm can keep the whole benefit of his innovation $\Delta$ in case when he is the sole innovator in downstream market. When both downstream firms are successful, competition dissipates part of the profit and each $D_j$ gets $\delta$. Thus the payoff to downstream firms is described as Table 1.

When $U_A$ and $D_1$ integrates (without loss of generality), no firm can do better than when they are separated. When only $D_1$ succeeds, he gets $\Delta$; when only $D_2$ succeeds, upstream competition leads to zero price for the input. When both downstream firms succeed, the integrated entity $U_A - D_1$ retrieves $\delta$ from its downstream affiliates; while competition leads to zero input price for $D_2$. Therefore, the payoff to $U_A - D_1$ and $D_2$ is again given by Table 1. We also get the same payoff matrix under full integration. Thus, we have the following result.

**Proposition 3.** When only downstream innovation is needed, downstream investment incentives are not affected by vertical integration, and social welfare is the same under vertical integration and vertical separation.

Compared to the case when only upstream innovation is needed, vertical integration has no effect on social welfare when only downstream innovation matters. This is simply because upstream competition leads to zero input price, which does not alter the nature of downstream competition. The difference between the situation when only upstream innovation is needed and when only downstream innovation is need originates from the fact that: the inputs produced by two upstream firms are homogenous and thus perfect substitutes for downstream firms; while downstream firms innovation are differentiated. From the perspective of the final product market, product differentiation is due to the innovation of downstream firms rather than that of the upstream firms. In other words, the upstream competition is stronger than the competition in the downstream market.

3.3 The General Case

The extreme divergence in the two benchmarks above results from the perfect substitution of upstream inputs. However, the general logic is that upstream competition is stronger than downstream competition, and vertical integration has a larger impact on upstream innovation than on downstream innovation.

Consider the general case where upstream inputs are also differentiated, but not channel specific. Specifically, if one upstream firm sells to both downstream firms, then each downstream firm gets profit $\delta_D$; if each upstream firm sells to one downstream firm, then each downstream firm gets profit $\delta_D + \delta_U$. Thus, $\delta_D$ measures downstream differentiation, and $\delta_U$ measures upstream differentiation. We prove the following result in the Appendix.
**Proposition 4.** If downstream is more differentiated than upstream, then vertical integration has larger impact on upstream innovation than downstream innovation; if upstream is more differentiated, then vertical integration has the same effect on both upstream and downstream innovation.

*Proof.* See Appendix.

In the special case where there is no upstream differentiation, we have the result as in our benchmarks: vertical integration only affects upstream investments but not downstream investments. The above proposition highlights two channels that vertical integration can affect investment incentives. The first channel is through better coordination inside the integrated entity. This is better understood when upstream differentiation is relatively weak. In this case, each firm can only get the benefit of differentiation which is contributed by his own product. Vertical integration allows each firm to also benefit from the differentiation that is created by his affiliate. Therefore, upstream firms benefit more since downstream firms are more differentiated.

The second channel is through bargaining. This is clearly shown in the situation when upstream is more differentiated. In this case, the firm that makes offer can obtain the value of differentiation from both upstream and downstream. However, the probability that a firm is going to make an offer is only 0.5. And thus vertical integration eliminates such bargaining friction, which improves investment incentives. Notice that even if upstream firms contribute to all the differentiation ($\delta_D = 0$), they cannot get the whole benefit of their innovation. This is due to the fact that downstream firms are sort of bottleneck, as they have the access to final consumers. Thus downstream firms are granted a stronger bargaining position. Moreover, since each downstream firm only needs one unit of input, they won’t compete for the same supplier; while upstream firms may want to compete against each other for selling to the same downstream firm. Hence, upstream competition is stronger than downstream competition, which makes upstream firms benefit more from vertical integration.

### 4 Interaction Between Upstream and Downstream Innovations

Based on the analysis of the previous two sections, we now study the situation when both upstream and downstream innovations are needed for the final product. This situation fits a number of settings: upstream firms may be manufactures who need to make investment in order to develop high quality product, while downstream retailers need to make investment in services in order to improve relationship with consumers; upstream firms may consist of patent holders who make investment to develop new ideas, while downstream developers invest to develop the ideas into final products. There is a strong complementarity between upstream and downstream innovations, in the sense that there is no value for the final product without either upstream innovation or downstream innovation.
Different from the two sections above, when there is only one upstream firm who succeeds in investment, upstream competition is not present and downstream investment incentives are different from what has been analyzed above. We start with the analysis in this situation. And we focus on the quadratic cost function in this section.

4.1 Downstream Investment with Upstream Monopolist

Given that only one upstream firm is successful, without loss of generality, suppose it is $U_A$. We first study the downstream investment incentives when all firms are separated.

4.1.1 Vertical Separation

When all firms are separated, in the continuation game each downstream firm chooses their investment independently. We make two remarks here. First, we maintain the time-line as before, i.e. the payoff to each firm is realized through ex post bargaining, which we follow the classic property right literatures. The downstream investment is not contractible, and the upstream monopolist cannot write a contract with downstream firms before any downstream investment happens. The bargaining takes place after the outcome of downstream investment is realized and observed by all firms.

Second, in the previous section, when only upstream innovation is needed for the final product, downstream competition is always present. Therefore, as long as the upstream monopolist makes successful innovation, she is able to extract the whole downstream market profit. When downstream innovation is needed for the final product, downstream competition is not guaranteed. When only one downstream firm obtains successful innovation, there is in fact an upstream monopolist and a downstream monopolist. Our simple bargaining procedure implies that in this case the upstream firm and the downstream firm each gets $\frac{\Delta}{2}$ (Each firm gets $\Delta$ when he is chosen to make offers, which happens with probability $\frac{1}{2}$). However, when both downstream firms succeed, downstream competition is again present, and the upstream monopolist is able to catch the whole downstream market profit $\Delta$ and each downstream firm ends up with zero profit. There is a major difference between our model and the model of Bolton and Whinston(1993): in their model, downstream competition is exogenously determined by whether there are one or two units of input available; however, in the present model, ex ante downstream competition always exists and ex post downstream competition is endogenously determined by the investment of each downstream firm.

The payoff matrix for the two downstream firms can be summarized as

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>0,0</td>
<td>$\frac{\Delta}{2}$,0</td>
</tr>
<tr>
<td>$F$</td>
<td>0,$\frac{\Delta}{2}$</td>
<td>0,0</td>
</tr>
</tbody>
</table>

Then for each downstream firm, they choose $d_j^*$ such that

$$d_j^* = \arg\max_{d_j} \{d_j(1 - d_{j'}')\frac{\Delta}{2} - C_D(d_j)\}, j' \neq j$$
The same reasoning as the previous section shows that under Assumption 2, there exists a unique equilibrium in the investment stage \( (d_1 = d_2 = d_{VS}^M) \) which is given by the solution to the following equation

\[
C_D'(d_{VS}^M) = (1 - d_{VS}^M) \frac{\Delta}{2}
\]

### 4.1.2 Vertical Integration

Now suppose that the upstream monopolist \( U_A \) integrates with one of the two downstream firms, without loss of generality, assume \( U_A \) and \( D_1 \) integrates. The game is the same as vertical separation except in the bargaining stage. First, when the independent downstream firm \( D_2 \) is the only one who succeeds in downstream innovation, the bargaining outcome is such that \( U_A - D_1 \) and \( D_2 \) each gets \( \Delta_2 \); Second, when \( D_1 \) is the sole innovator in downstream market, the integrated entity \( U_A - D_1 \) retains the whole downstream market profit \( \Delta \). And thus \( D_1 \) is able to get the whole benefit of his investment. Third, when both downstream firms succeed, there is no actual downstream competition. The upstream monopolist only supplies her downstream affiliates. The payoff matrix is given by

<table>
<thead>
<tr>
<th></th>
<th>( U_A - D_1 )</th>
<th>( D_2 )</th>
<th>S</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>( \Delta, 0 )</td>
<td>( \Delta, 0 )</td>
<td>F</td>
<td>( \Delta ), ( \frac{\Delta}{2} )</td>
</tr>
<tr>
<td>F</td>
<td>( \Delta ), ( \frac{\Delta}{2} )</td>
<td>0, 0</td>
<td>F</td>
<td>( \Delta ), ( \frac{\Delta}{2} )</td>
</tr>
</tbody>
</table>

Then under vertical integration, the problem for \( U_A - D_1 \) and \( D_2 \) is to choose \( d_1^* \) and \( d_2^* \) such that

\[
d_1^* = \text{argmax}_{d_1} \{d_1 \Delta + (1 - d_1) \frac{\Delta}{2} - C_D(d_1)\}
\]

and

\[
d_2^* = \text{argmax}_{d_2} \{d_2 (1 - d_1) \frac{\Delta}{2} - C_D(d_2)\}
\]

The unique equilibrium \( (d_{VI}^1, d_{VI}^2) \) in the investment stage under Assumption 2 is given by the solution to the following equations

\[
C_D'(d_{VI}^1) = \Delta - \frac{1}{2} \Delta d_{VI}^2
\]

\[
C_D'(d_{VI}^2) = \frac{\Delta}{2} - \frac{1}{2} \Delta d_{VI}^1
\]

**Lemma 2.** The integrated downstream firm invests more than the independent downstream firm, indeed we have \( d_{VI}^1 > d_{VS}^M > d_{VI}^2 \).

**Proof.** The proof is similar to the proof of Proposition 1, we thus omit the detailed proof.

Lemma 2 can be shown in the Figure 2. The best response function for the independent downstream firm \( D_2 \) is not affected by vertical integration. However, the best response function is pushed outward by vertical integration. There are two effect of vertical integration on downstream investments, first, the integrated downstream firm is able to catch all the benefit of his innovation whenever he is successful, no matter whether the independent downstream firm succeeds or not. This boosts the investment incentive of the integrated firm. Second, even
if the integrated downstream firm fails in innovation, the integrated entity $U_A - D_1$ still gets $\Delta$ if the independent downstream firm succeeds, which tends to decrease the investment incentive of the integrated downstream firm. Starting from the symmetric equilibrium as in the vertical separation case, the net effect of vertical integration is to increase the investment incentive of the integrated downstream while leave that of the independent downstream firm unchanged. Since downstream investments are strategic substitutes, higher investment incentive from the integrated downstream firm crowds out the incentive of the independent firm.

Figure 3: Equilibrium Downstream Investment under Vertical Separation and Vertical Integration

Lemma 3. If there is only one successful upstream firm, in the continuation game, the profit for the integrated upstream monopolist is higher than the separated one.

Proof. See Appendix.

Under vertical separation, there is serious under-investment problem: each downstream firm can only get half of the benefit ($\Delta/2$) from his innovation even when he is the sole innovator in the downstream market, while the social benefit in this case is at least $\Delta$. Under vertical integration, the insufficient incentive problem persists for the independent downstream firm; however, the hold-up problem is solved for the integrated entity. This increases the profit of the integrated upstream monopolist. Moreover, the integrated entity gets additional benefit from the crowd-out of the investment from the independent downstream firm. A key message is that, opposed to the benchmark when only upstream innovation matters, the upstream monopolist is not able to get the whole industry profit. And thus, there is potentially large gain from vertical integration, both privately and socially, since excessive investment incentive is not an serious concern any more.

4.2 Upstream Investment—Vertical Separation

Now we turn to the investment incentives of upstream firms when all firms remain separated. When only one upstream firm succeeds, the subgame goes as in Section 4.1.1, the upstream
monopolist gets continuation payoff $\pi_{VS}$. When both upstream firms obtain successful innovation, the subgame goes as Section 3.2, where each upstream firm gets zero continuation payoff. Therefore, the payoff matrix for the upstream firms at the investment stage is given by

<table>
<thead>
<tr>
<th></th>
<th>$U_A$</th>
<th>$U_B$</th>
<th>S</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0,0</td>
<td>$\pi_{VS}0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>$0,\pi_{VS}$</td>
<td>0,0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Under Assumption 1, there is a unique equilibrium $(e_{VS}, e_{VS})$ in the investment game which is given by the solution to the following equation,

$$C'_U(e_{VS}) = (1 - e_{VS})\pi_{VS}$$

### 4.3 Upstream Investment–Vertical Integration

Suppose now $U_A$ and $D_1$ integrate. When the independent upstream firm $U_B$ is the only one who succeeds in upstream innovation, the subgame goes as Section 4.1.1, where the profit for $U_B$ is $\pi_{VS}$. However, in this circumstance, even though $U_A$ does not have a successful upstream innovation, she can still get positive profit from her downstream affiliate when $D_1$ is the sole downstream innovator. We denote this profit as $\pi_{FI}^F$.

When the integrated upstream firm $U_A$ is the sole upstream innovator, the subgame goes as Section 4.1.2. The profit for the upstream monopolist $U_A$ is given by $\pi_{VI}$. As the previous lemma shows, we have $\pi_{VI} > \pi_{VS}$.

When both upstream firms make successful investment, the subgame goes as Section 3.2. However, the profit for the two upstream firms are different: the independent upstream firm $U_B$ gets zero profit due to competition from $U_A$; while the integrated firm can still get positive profit by supplying her downstream affiliates. We denote this profit as $\pi_{VI}^D$.

Then the payoff matrix for upstream firms in the investment stage is given by

<table>
<thead>
<tr>
<th></th>
<th>$U_A$</th>
<th>$D_1$</th>
<th>$U_B$</th>
<th>S</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>$\pi_{VI}^D$</td>
<td>$\pi_{VI},0$</td>
<td>$\pi_{VS},0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>$\pi_{VI},\pi_{VS}$</td>
<td>0,0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Then the unique equilibrium in the upstream investment stage is given by

$$\begin{cases} 
C'_U(e_{VI}^1) = \pi_{VI} - (\pi_{VI} + \pi_{VI}^F - \pi_{VI}^D)e_{VI}^2 \\
C'_U(e_{VI}^2) = \pi_{VS} - \pi_{VS}e_{VI}^1
\end{cases}$$

then the following proposition can be easily shown

**Proposition 5.** The integrated upstream firm invests more than the independent upstream firm, indeed we have $e_{VI}^1 > \pi_{VS} > e_{VI}^2$. 

16
Proof. See Appendix.

The effect of vertical integration is three-fold: first, there is better coordination inside the integrated entity \( U_A - D_1 \). This is the case of our benchmark in Section 3.1. When both upstream firms have successful innovation, they only compete for the independent downstream firm. This increases the investment incentive for the integrated upstream firm and decreases the incentive of the independent upstream firm.

Second, the elimination of hold-up problem between \( U_A \) and \( D_1 \) further increases the investment incentive of the integrated upstream firm, which results from the fact that \( \pi_{VI} > \pi_{VS} \). This effect is only present when downstream innovation is also needed for the product: an integrated upstream monopolist has larger continuation payoff than an independent upstream monopolist. In this sense, downstream innovation amplifies the crowd-out effect of vertical integration on the independent upstream firm.

Third, the combination of upstream innovation and downstream innovation gives rise to an additional effect. This originates from the fact that the integrated upstream firm obtains positive profit even when he fails in upstream investment, since the downstream affiliate \( D_1 \) still gets a profit of \( \Delta/2 \) when he is the only successful downstream innovator. This tends to reduce the investment incentive of the integrated upstream firm. The above proposition shows that this effect is dominated, and the overall effect of vertical integration is a result of crowd-out of the investment the independent upstream firm.

4.4 Welfare Implication

As in the previous section, we embed our welfare analysis in the simple Hotelling framework.

Denote \( W_D \) as the social welfare in the continuation game when both upstream firms succeed in investment; similarly, denote \( W_{VS}^M \) and \( W_{VI}^M \) as the social welfare in the continuation game when there is only one successful upstream innovator, who is vertically separated or vertically integrated respectively. And thus the total social welfare under vertical separation is given by

\[
W_{VS} = e_{VS}W_D + 2e_{VS}(1 - e_{VS})W_{VS}^M - 2C_U(e_{VS})
\]

and the social welfare under vertical integration is given by

\[
W_{VI} = e_{VI}W_D + e_{VI}(1 - e_{VI})W_{VI}^M + e_{VI}(1 - e_{VI})W_{VS}^M - C_U(e_{VI}) - C_U(e_{VI}^2)
\]

As in the last section, with \( v = c_u = c_d = 1 \), we can show that vertical integration always increases social welfare, as depicted in Figure 4.\(^2\)

The main source of the positive welfare effect of vertical integration is the resolution of the hold-up problem between the integrated upstream and downstream firms. This increases social welfare in two ways: Firstly, the social welfare is higher when the upstream monopolist is vertically integrated than when it is vertically separated. Thus, the upstream crowd-out effect means that it is more likely to be the integrated upstream firm that is going to be

\(^2\)Further calculation shows that this property holds for any product differentiation \( t \in [0, 0.5] \), and investment cost \( c_u, c_d \geq 1 \).
the sole innovator in the upstream market. Secondly, the total probability that there will be at least one successful product in the market is larger. This is because the total amount of investment in both upstream and downstream level increases.\(^3\) In a word, when both upstream and downstream innovations are important for the final product, the investment incentives are generally insufficient due to the hold-up problem. Vertical integration partially overcomes this problem and pushes the investment levels towards social optimum.

### 4.5 Full Integration

It is clear from the above analysis that, the integration of \(U_A\) and \(D_1\) hurts both the independent upstream firm \(U_B\) and the independent downstream firm \(D_2\). Hence \(U_B\) may also have incentive to integrate with the independent downstream firm \(D_2\). There are indeed joint gains from such integration: first, when \(U_B\) is the only upstream innovator, they jointly get higher profit since the investment of \(D_1\) will be crowded out by \(D_2\) now. Second, when both upstream firms succeed or only \(U_A\) succeeds, the joint profit of \(U_B\) and \(D_2\) does not change. Therefore, \(U_B\) and \(D_2\) indeed have incentives to integrate. Moreover, full integration further increases social welfare as shown in Figure 5.

The further welfare improvement from full integration comes from two effects: first, hold-up problem is now totally resolved inside each integrated entity, which increases welfare; Second, the crowd-out effect of partial vertical integration is eliminated, which helps reduce the total investment cost.

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\(^3\)This is because the best response curves have slope less than 1, and thus the reductions in investments from independent firms do not fully offset the increases from the integrated firms.

Figure 4: Welfare Effect of Vertical Integration with Both Upstream and Downstream Innovations
5 Discussion and Extension

5.1 Information Disclosure by Upstream Firms

In our model, we assume that a final product necessitates both upstream and downstream innovation; but upstream innovation and downstream innovation are independent in the sense that the downstream innovation does not require any information or actual delivery of upstream innovation. All that downstream firms need to know is whether there is successful upstream innovation or not. In this subsection, we relax this assumption and assume that downstream firms need information about the upstream innovation in order to make successful investment. We focus on quadratic cost function.

We modify the game as follows,

- **Upstream Investment Stage**: Each $U_i$ makes investment decision $e_i$; if both firms fail, the game ends;

- **Information Disclosure Stage**: The successful upstream firm decides whether to disclose the information about the innovation to both downstream firms or only one of them;

- **Downstream Investment Stage**: Each $D_j$ makes investment decision $d_j$ if he receives information from the upstream firm; if both firms fail, the game ends;

- **Bargaining Stage**: The successful upstream firm(s) and successful downstream firm(s) bargain over the price of the input;

- **Payoff Stage**: Payments are made and inputs are delivered if any agreement is reached; downstream market realizes and game ends.

In the game above, downstream investment needs information about the upstream innovation but not the actual delivery of the input. In other words, if both upstream firms have successful innovation, the downstream firm is free to choose any upstream supplier no matter
from where he gets the necessary information for investment. We assume that when the up-
stream firm is indifferent between disclose information and not disclose any information, he
chooses to disclose. Then the subgame is the same as in Section 3 if both upstream firms make
successful investment under either vertical separation or vertical integration. Because even
though the integrated upstream firm may refrain from disclosing information to the indepen-
dent downstream firm, the upstream competitor would disclose such information.

When it turns out to be the case that there is a monopolistic upstream innovator, the
incentive to disclose may differ depending on whether the upstream monopolist is vertically
integrated or not. Under vertical separation, the profit for the upstream monopolist is \( \pi_{VS} \) if he
discloses the information to both downstream firms; when he only discloses to one downstream
firm, it is easy to know that downstream investment is \( d = \frac{\Delta}{2cd} \), and the profit for the upstream
monopolist is \( \pi_{1VS} = \frac{\Delta^2}{4cd} \).

When the upstream monopolist is vertically integrated with one of the two downstream
firms, the profit is \( \pi_{VI} \) if he also discloses the information to the independent downstream firm.
When he refrains from disclosing the information, downstream investment is given by \( d = \frac{\Delta}{cd} \)
and the profit for the integrated upstream monopolist is \( \pi_{1VI} = \frac{\Delta^2}{2cd} \). Suppose there is a cost \( K \)
related to disclosing information to a second downstream firm. Such cost may be related to
the risk of information leakage, where it is only a private cost but not social cost; or the cost
may be about how to convey the information correctly to the downstream firms, then it is also
a social cost. To show our main insight, we assume such cost is only a private cost. The next
proposition shows that an integrated upstream firm has less incentive to disclose information
to both downstream firms.

**Proposition 6.** There exists a range of value \( K \in (K, \bar{K}) \) such that the separated upstream
monopolist discloses information to both downstream firms, while the integrated upstream mo-
nopolist does not disclose to the independent downstream firm.

**Proof.** See Appendix.

Under vertical separation, the upstream monopolist has more incentive to disclose infor-
mation to a second downstream firm. First, due to our assumptions on the cost function,
downstream competition does not lower the total level of downstream investment; second,
when both downstream firms obtain successful innovation, the payoff for the upstream monop-
olist is now \( \Delta \) rather than \( \frac{\Delta}{2} \). However, under vertical integration, by disclosing information
to the independent downstream firm, the integrated upstream firm has to balance the benefit
of lowering investment cost (since now she makes less investment) and the loss of profit when
the independent downstream firm is the only downstream innovator (since now the integrated
firm can only get \( \frac{\Delta}{2} \) rather than \( \Delta \)).

With respect to welfare, if \( K \in (K, \bar{K}) \), the social welfare is the same as in the previous
section under vertical separation. However, the result is different under vertical integration
when the integrated upstream firm is the only upstream innovator. Since in this case, the
integrated upstream firm would choose to only disclose information to her downstream affiliate.
It can be easily shown that if downstream investment cost is relatively high, the welfare under
vertical integration is lower than that under vertical separation if there is only one upstream innovator.

**Proposition 7.** When there is only one upstream innovator, if \( K \in (\bar{K}, \bar{K}) \), there exists a \( \bar{c}_d > \Delta \) such that social welfare is higher under vertical separation than vertical integration if downstream investment cost is large enough, i.e. if \( c_d > \bar{c}_d \).

**Proof.** See Appendix.

Therefore, when downstream investment cost is low, the benefit from the elimination of hold-up problem inside the integrated entity outweighs the cost from excluding the independent downstream firm; however, when downstream cost is high, vertical separation delivers higher social welfare as now promoting investment incentive becomes the main issue. Taken into consideration upstream innovation as well, the result still holds. Social welfare is higher under vertical integration if downstream investment cost is low, and higher under vertical separation otherwise.

### 5.2 Ex ante Bargaining

In the discussion above, we assume that bargain between upstream firms and downstream firms happens after all outcomes of investments have already realized and been observed by all firms. The main insights in our paper still hold if bargain happens in an ex ante stage, i.e. upstream firms bargain with downstream firms after the outcomes of upstream investments realized but before downstream firms make any investment. This would be the case if downstream innovation needs the actual delivery of the input. In this situation, hold-up problem does not exist for downstream firms; however vertical integration still affects the investment incentives of upstream firms.

First, the upstream competition soften effect still exists. When both upstream firms obtain successful innovation, they only compete for the independent downstream firm. Thus the integrated upstream firm gets positive profit while the independent upstream firm gets zero profit.

Second, when only one of the two upstream firms succeeds, the integrated upstream monopolist is able to catch a larger part of downstream market profit even though now the bargaining does not affect downstream investment incentives. This results from two effects: on one hand, the integrated upstream monopolist only bargains with the independent downstream firm; on the other hand, the integrated upstream firm holds a stronger bargaining position than the independent upstream firm, since the outside option for the integrated upstream firm is higher. As before, this two factors tend to induce overall over-investment and reduce social welfare. Since there is no gain from the elimination of hold-up problem, the net effect of vertical integration on social welfare would generally be negative.

Third, when the upstream monopolist faces the choice between selling to both downstream firms and selling to only one of them, efficient bargaining implies that a separated upstream monopolist has the same incentive as an integrated upstream one. They would sell to both downstream firms only if downstream market profit is higher when there are two firms than
when there is only one, since vertical integration has no impact on downstream investment anymore. However, this result depends on the exact form of bargaining. For instance, if the upstream monopolist have all the bargaining power, i.e. she makes take-it-or-leave-it offers to each downstream firm, then if the offer is not publicly observable or the offer is sequentially made to downstream firms, lack of commitment problem may lead to inefficient trading under vertical separation. There would be too much trading since the upstream monopolist cannot commit to trade with only one downstream firm. However, such problem does not exist under vertical integration, because the integrated upstream monopolist will always sell to the downstream affiliate. And thus she would only trade with the independent downstream firm when it is efficient to have both downstream firms active.

Furthermore, whether downstream market profit is higher when both downstream firms are active or when only one firm is active depends on the downstream investment cost. Such downstream investment cost may be affected by upstream innovation; for instance, successful upstream innovation decreases downstream investment cost. When the outcome of upstream innovation is not observable, or when information about such cost is only known for the upstream innovator, the upstream innovator may be able to signal her success by restricting trading with downstream firms.

6 Conclusion

In this paper, we studied the effect of vertical integration on the investment incentives of upstream and downstream firms. When only upstream innovation is needed for the industry, vertical integration may lead to overall over-investment and decreases social welfare. The crowd-out effect of vertical integration is strengthened when downstream innovation is also needed. However, vertical integration also promotes both upstream and downstream investment. The overall impact of vertical integration on social welfare turns out to be positive. Our results suggest that when evaluating the impact of vertical integration, especially in industries with intensive innovation, the exact nature between upstream and downstream investment may be a key point in the decision. Studying the impact of vertical integration in a more general bargaining environment, or in the presence of other forms of complementarity between upstream and downstream innovation might be interesting avenues for future research.

7 Appendix

7.1 Proof of Lemma 1

Clearly, when both upstream firms fail in investment, they both get zero profit. When at least one upstream firm succeeds, we distinguish between two cases: when only one succeeds who then acts as an upstream monopolist; and when both upstream firms succeed and upstream competition emerges.

Case 1: Only one upstream firm succeeds, suppose it is $U_A$. 
If it is $U_A$ who is chosen to make offers, then $U_A$ can always guarantee a payoff of $\Delta$ by making offers $(\Delta, E)$ and $p(p > \Delta)$ to the two downstream firms respectively, which will be accepted by the first one and rejected by the second one. And thus $U_A$ gets profit $\Delta$, which is the maximum profit he can extract.

If it is the downstream firms that were to make offers, $U_A$ can again get payoff $\Delta$, since competition between the two downstream firms would drive the price up to $\Delta$. To see this, since the outside option for all firms is zero, $U_A$ would accept any offer with non-negative price. For downstream firms $D_1$ and $D_2$:

First, any offer $\{(p_1), (p_2)\}$ without an exclusive dealing clause cannot be an equilibrium. To see this, any such offer with non-negative price would be accepted by $U_A$, since the maximum profit a downstream firm can get when both downstream firms are active in the downstream market is $\delta$, it will never be rational for either downstream firm to make a non-exclusive dealing offer with price higher than $\delta$. Moreover, any offer $\{(p_1), (p_2)\}$ where $p_i \leq \delta$ cannot be an equilibrium either, since either firm would then deviate by proposing an exclusive dealing offer $\{p_1 + p_2, E\}$, which will be accepted by $U_A$ and the deviating firm gets larger profit since $\Delta > 2\delta$. And hence there is no equilibrium where both firms propose non-exclusive dealing offers.

Second, there must be at least one firm who proposes an offer with price $\Delta$ in equilibrium. The above analysis shows that there must be at least one downstream firm who propose an exclusive dealing offer, which means that $U_A$ can only accept one offer from the downstream firms. If both offers have price lower than $\Delta$, then the firm who proposed the lower price offer, say $D_1$, can deviate by proposing $p_2 < p_1 < \Delta$ and earn positive profit rather than zero. And hence the maximum price of the two offers must be $\Delta$, and $U_A$ would accept such offer and get profit $\Delta$.

Thus when only $U_A$ succeeds in investment, no matter who makes the offer, $U_A$ is able to extract the industry profit $\Delta$.

Case 2: Both upstream firms succeed in investment.

If the downstream firms make offers, they will ask for zero price to both upstream firms. And no downstream firm can be better off by making other offers. The reason is simple: the upstream firm is willing to accept any non-negative offers, then any positive offer without exclusive dealing clause cannot make the downstream firm better off; moreover, any acceptable exclusive dealing offer cannot exclude the other downstream firm from the market. Therefore, there is no profitable deviation for the downstream firm. And the profit for each upstream firm is zero. If the upstream firms make offers, the best they can do is to offer zero price, since otherwise the other upstream firm will always undercut.

Therefore, when both upstream firms are successful, they each get zero profit under vertical separation.
7.2 Proof of Proposition 1

Rewrite Equation (1) as

\[
\begin{align*}
C'_U(e^1) &= \Delta(1 - e^2) + \alpha e^2 \\
C'_U(e^2) &= \Delta(1 - e^1)
\end{align*}
\]

When \( \alpha = 0 \), the solution corresponds to the investment level under vertical separation; when \( \alpha = \delta \), it is the solution under vertical integration. It is clear to see that

\[
\frac{\partial e^1}{\partial \alpha} = e^2 C''_U(e^1)
\]

which is always positive under Assumption 1. Therefore, we must have \( e^1_{VI} > e^*_{VS} \). Furthermore, we have \( C'_U(e^2_{VI}) = \Delta(1 - e^1_{VI}) < \Delta(1 - e^*_{VS}) = C'_U(e^*_{VS}) \), and thus \( e^2_{VI} < e^*_{VS} \).

7.3 Proof of Proposition 2

As \( v = 1 \), we focus on the case where \( t \in [0, 0.5] \). With quadratic cost function, under vertical separation, upstream investment is given by

\[
e_{VS} = \frac{\Delta}{c + \Delta} = \frac{1 - t}{2 - t}
\]

the corresponding social welfare is given by

\[
w_{VS} = e_{VS}^2(1 - \frac{t}{4}) + 2e_{VS}(1 - e_{VS})(1 - \frac{t}{2}) - e_{VS}^2 = \frac{1}{(4t - 2)^2}(-t^3 + 6t^2 - 13t + 8)
\]

Under vertical integration, upstream investments are

\[
e^1_{VI} = \frac{\Delta(c_u - (\Delta - \delta))}{c_u^2 - \Delta(\Delta - \delta)} = \frac{(1 - t)\frac{3t}{2}}{1 - (1 - t)(1 - \frac{3t}{4})}
\]

and

\[
e^2_{VI} = \frac{\Delta(c_u - \Delta)}{c_u^2 - \Delta(\Delta - \delta)} = \frac{(1 - t)t}{1 - (1 - t)(1 - \frac{3t}{4})}
\]

the corresponding social welfare is then

\[
w_{VI} = e^1_{VI}e^2_{VI}(1 - \frac{t}{4}) + (e^1_{VI}(1 - e^2_{VI}) + e^2_{VI}(1 - e^1_{VI}))(1 - \frac{t}{2}) - \frac{1}{2}e^1_{VI}^2 - \frac{1}{2}e^2_{VI}^2
\]

\[
= \frac{1}{(2(3t - 5)^2)}(-6t^4 + 27t^2 - 46t + 25)
\]

We have

\[
w_{VS} - w_{VI} = \frac{t}{4(3t^2 - 11t + 10)^2}(3t^4 - 18t^3 + 34t^2 - 22t + 3)
\]

The solution of the above equation tells us that

\[w_{VS} - w_{VI} > 0\] if \( 0 < t < \hat{t} \)

and

\[w_{VS} - w_{VI} < 0\] if \( \hat{t} < t < 0.5 \)

where \( \hat{t} = 1 - \frac{\sqrt{6}}{3} \approx 0.18. \)
7.4 Proof of Proposition 4

To see the effect of vertical integration, we need to determine the payoff-matrix for the firms that make investments.

In both cases (when only upstream innovation is needed or only downstream innovation is needed), if both firms fail in investment, the payoffs for all firms are zero. Similarly, if only one firm succeeds in investment, the firm is able to extract the industry profit $\Delta$.

The situation is different if both firms are successful, where we have bilateral duopoly. For simplicity, we assume that when $U_A(D_1)$ is indifferent between the offers of $D_1(U_A)$ and $D_2(U_B)$, $U_A(D_1)$ chooses the offer of $D_1(U_A)$. Similar assumption applies for $U_B$ and $D_2$. Moreover, we assume that when multiple equilibria exist, the two firms that make offers coordinate on the most profitable one.

The case when it is the downstream firms who make offers is simple. Each downstream firm offers $p = 0$ to both upstream firms is an equilibrium, and each downstream firm gets profit $\delta_D + \delta_U$. There is no profitable deviation for either downstream firm. The only way to get higher profit than $\delta_D + \delta_U$ is to make offers such that the other downstream firm is excluded from the market. However, this is not possible since the upstream firms are willing to take any non-negative offers from the downstream firms, and one downstream firm cannot make exclusive dealing offers to both upstream firms. This is not the only equilibrium, however no other equilibrium gives higher profit to the downstream firms.

Now we turn to the situation when it is the upstream firms who make offers. Denote $O_{ij}$ as the offer made by upstream firm $U_i, i = A, B$ to downstream firm $D_j, j = 1, 2$. We focus on a particular type of equilibrium, which we call symmetric equilibrium. A symmetric equilibrium is such that $O_{A1} = O_{B2}$, and $O_{A2} = O_{B1}$.

**Case 1:** $\delta_U \leq \delta_D$

This is the case when downstream is more differentiated than upstream. First, we analyze the equilibrium when all offers are simply prices without exclusive dealing clause. Suppose $(p_1, p_2)$ is offered by $U_A$ in equilibrium, then by symmetry $U_B$ offers $(p_2, p_1)$.

**Claim 1:** In equilibrium, the two downstream firms choose different suppliers, and $p_1 = p_2 + \delta_U$.

To see the first part, w.l.o.g, suppose both downstream firms choose $U_A$. Then we must have $\delta_D - p_1 \geq \delta_D + \delta_U - p_2$, and $\delta_D - p_2 > \delta_D + \delta_U - p_1$, which are contradictory. And thus the two downstream firms choose different suppliers. Assuming that in equilibrium $U_A$ supplies $D_1$, and $U_B$ supplies $D_2$. Then we must have $\delta_D + \delta_U - p_1 \geq \delta_D - p_2$, which gives us $p_1 - p_2 \leq \delta_U$. Profit maximization for $U_A$ means that we must have $p_1 = p_2 + \delta_U$.

According to Claim 1, the offer of $U_A$ can be simplified to $(p + \delta_U, p)$, similarly for $U_B$. Then each upstream firm gets a profit of $p + \delta_U$, and each downstream firm gets a profit of $\delta_D - p$.

We have the following result.

**Claim 2:** In equilibrium we must have $p \leq \delta_U$.

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As what makes clear later, firms would maximally differentiate themselves in equilibrium. And thus the two downstream firms won’t choose the same supplier in equilibrium. Our notion of symmetric equilibrium means that the two downstream firms receive the same offer from their suppliers.

This simplification does not matter up to a relabelling of firms.
First, any \( p > \delta_D \) cannot be an equilibrium, as the profit of the downstream firms will be negative. Suppose \( \delta_D \geq p > \delta_U \), given the offer of \( U_B \) as \((p, p + \delta_U)\), by offering \((p - \frac{\epsilon}{2}, p - \frac{\epsilon}{2})\) to both firms, \( U_A \) can get a profit of \( 2p - \epsilon > p + \delta_U \). To see this, accepting this offer from \( U_A \), each downstream firm gets a profit of \( \delta_D - (p - \frac{\epsilon}{2}) > \delta_D - p \).

Then we can show that any \( 0 \leq p \leq \delta_U \) can be an equilibrium. Given that \( U_B \) offers \((p, p + \delta_U)\), the above argument implies that \( U_A \) cannot be better off by attracting both downstream firms if \( p \leq \delta_U \). Moreover, \( U_A \) cannot extract more than \( p + \delta_U \) from \( D_1 \), since otherwise \( D_1 \) would accept the offer of \( U_B \). If \( U_A \) tries to attract \( D_2 \), he has to offer a price lower than \( p + \delta_U \). Hence, there is no profitable deviation for \( U_A \).

By our assumption that the two upstream firms can coordinate on the most profitable equilibrium, there is an equilibrium in which \( U_A \) offers \((p_{A1} = 2\delta_U, p_{A2} = \delta_U)\) and \( U_B \) offers \((p_{B1} = \delta_U, p_{B2} = 2\delta_U)\). Each upstream firm gets profit \( 2\delta_U \), and each downstream firm gets profit \( \delta_D - \delta_U \).

Now suppose the upstream firms make offers with exclusive dealing clause, and we consider a symmetric equilibrium as above. The presence of exclusive dealing clause implies that the two downstream firms much choose different suppliers. Moreover, a symmetric equilibrium where \( U_A \) offers \(((p_1, E), (p_2, E))\) will be equivalent to offer \(((p_1, E), p_2)\), since \( U_A \) will only serve one downstream firm. We consider the former type of offer.

**Claim 3:** In equilibrium, we have \( p_1 = p_2 \leq 2\delta_U \).

First, it is to see that any \( p_1 > \delta_D + \delta_U \) or \( p_2 > \delta_D + \delta_U \) cannot be an equilibrium, as such offer will never be accepted by either downstream firm. Second, \( p_1 \neq p_2 \) cannot be an equilibrium either, since the downstream firms will accept the offer with lower price. Therefore, we must have \( p_1 = p_2 = p \leq \delta_D + \delta_U \).

However, any \( 2\delta_U < p \leq \delta_D + \delta_U \) cannot be an equilibrium. Given that \( U_B \) offers \(((p, E), (p, E))\), by offering each downstream firm a price \( p' = p - \delta_U - \frac{\epsilon}{2} \), \( U_A \) gets a profit of \( 2p - 2\delta_U - \epsilon > p \). And both downstream firms are willing to accept this offer, since the profit from accepting is \( \delta_D - p' = \delta_D + \delta_U - p + \epsilon > \delta_D + \delta_U - p \). Therefore, we must have \( p \leq 2\delta_U \) in equilibrium.

Hence, when we allow upstream firms to make offers with exclusive dealing clause, the profit maximization equilibrium gives each upstream firm a profit of \( 2\delta_U \), and each downstream firm gets \( \delta_D - \delta_U \).

**Case 2:** \( \delta_U > \delta_D \)

As the first case, it cannot be the case that both downstream firms choose the same supplier, with either simple price offers or offers with exclusive dealing clause. In an equilibrium with simple price offers, we can focus on such situation that \( U_A \) offers \((p + \delta_U, p)\), where \( p \leq \delta_D \). We show that \((\delta_D + \delta_U, \delta_D)\) is an equilibrium and it is the profit maximizing equilibrium for the upstream firms. First, no upstream firm can do better since there is no way to exclude one downstream firm from the market. Moreover, no upstream firm has profitable deviation. Given that the offer of \( U_B \) is \((\delta_D, \delta_D + \delta_U)\), decreasing either price cannot make \( U_A \) better off. Increasing the offer to \( D_1 \) will lose this downstream firm. Increasing the offer to \( D_2 \) has no effect. Similarly, by making exclusive dealing offer to one of the two downstream firms cannot make \( U_A \) better off.

Similar argument as in the first case indicates that there is also an equilibrium with exclusive
dealing clause where each upstream firm offer \((\delta_D + \delta_U, E)\) to each downstream firm. In a word, when the upstream firms make offers in this second case, the upstream firm each gets a profit of \(\delta_D + \delta_U\), and each downstream firm gets zero profit.

Now we turn to the effect of vertical integration on investment incentives. When downstream firms are more differentiated than upstream firms (Case 1), if only upstream innovation is needed, the payoff matrix is shown Table 9. It is clear that the effect of vertical integration on upstream investment incentives can be measured by \(\delta_D\).

<table>
<thead>
<tr>
<th>Table 8: Only Upstream Innovation is Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Vertical Separation</td>
</tr>
<tr>
<td>(U_A U_B)</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td>F</td>
</tr>
</tbody>
</table>

If only downstream innovation is needed, the payoff matrix is given by Table 10. And the effect of vertical integration on downstream investment can be measured by \(\delta_U\). Since in Case 1, \(\delta_U \leq \delta_D\), the effect of vertical integration has a larger effect on upstream innovation than on downstream innovation. In the special case where \(\delta_U = 0\) as in our benchmarks, vertical integration only affects upstream investments but not downstream investments.

<table>
<thead>
<tr>
<th>Table 9: Only Downstream Innovation is Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Vertical Separation</td>
</tr>
<tr>
<td>(D_1 D_2)</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td>F</td>
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</tbody>
</table>

When upstream firms are more differentiated than downstream firms (Case 2), the payoff matrix for the innovating firms is given by the same Table 11, no matter when only upstream innovation is needed or only downstream innovation is needed. And vertical integration has the same effect on upstream investments and downstream investments.

<table>
<thead>
<tr>
<th>Table 10: When Upstream is More Differentiated</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Vertical Separation</td>
</tr>
<tr>
<td>(D_1(U_A) D_2(U_B))</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td>F</td>
</tr>
</tbody>
</table>

7.5 Proof of Lemma 3

With quadratic cost function, we have

\[
d_{VS}^M = \frac{\Delta}{2c_d + \Delta}
\]
under vertical separation; and under vertical integration we have
\[ d_{VI}^1 = \frac{\Delta (4c_d - \Delta)}{4c_d^2 - \Delta^2} \text{ and } d_{VI}^2 = \frac{2\Delta (c_d - \Delta)}{4c_d^2 - \Delta^2} \]
Thus,
\[ \pi_{VS} = \frac{\Delta^2}{2c_d + \Delta} \]
and
\[ \pi_{VI} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2} (12c_d^3 - 8c_d^2\Delta - \frac{1}{2}c_d\Delta^2 + \Delta^4) \]
Therefore, after simplification, we have
\[ \pi_{VI} - \pi_{VS} = \frac{\Delta^2}{4c_d^2 - \Delta^2} (4c_d^3 + \frac{3}{2}c_d\Delta^2 - 4c_d^2\Delta) \]
which is always positive since \( c_d \geq \Delta \).

### 7.6 Proof of Proposition 5

The best response function for the independent upstream firm is unchanged after integration. The best response function for the integrated upstream firm is given by
\[ C'_{VI}(e_{VI}) = \pi_{VI} - (\pi_{VI} + \pi_{VI}^F - \pi_{VI}^D)e_{VI}^2 \]
which can be obtained from an outward shift and rotation of the best response function under vertical separation. Then to prove the result, the only thing we need to show is that \( \pi_{VI}^F < \pi_{VI}^D \), which guarantees that the rotation does not offset the effect of outward shift. With quadratic cost function, we have
\[ \pi_{VI}^F = \frac{1}{2}c_d^2 \left( \frac{\Delta}{2c_d + \Delta} \right)^2 \]
and
\[ \pi_{VI}^D = \frac{1}{2}c_d\left( \frac{\Delta}{c_d + \Delta - \delta} \right)^2 \]
Clearly we have \( \pi_{VI}^F < \pi_{VI}^D \), since \( 2c_d + \Delta > c_d + \Delta - \delta \).

### 7.7 Proof of Proposition 7

It suffices to show that \( \pi_{VS} - \pi_{VS}^1 > \pi_{VI}^S - \pi_{VI}^1 \), which is equivalent to show \( \pi_{VI}^1 - \pi_{VS}^1 > \pi_{VI}^S - \pi_{VI} \). With quadratic cost function, we have
\[ \pi_{VI}^1 - \pi_{VS}^1 = \frac{\Delta^2}{4c_d} \]
and
\[ \pi_{VI}^S - \pi_{VS} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2} (16c_d^3\Delta + \Delta^4 - 14c_d^2\Delta^2) \]
After simplification, we have
\[ (\pi_{VI}^1 - \pi_{VS}^1) - (\pi_{VI}^S - \pi_{VS}) = \frac{\Delta^2}{4c_d(4c_d^2 - \Delta^2)^2} (16c_d^3\Delta + \Delta^4 - 14c_d^2\Delta^2) \]
which is always positive since \( c_d > \Delta \).

Let \( K = \pi_{VI}^S - \pi_{VI}^1 \) and \( \bar{K} = \pi_{VS} - \pi_{VS}^1 \), the for \( K \in (K, \bar{K}) \), we have \( \pi_{VS} - \pi_{VS}^1 - K > 0 \), while \( \pi_{VI}^1 - \pi_{VI} - K < 0 \).
7.8 Proof of Proposition 8

With quadratic cost function, when only one upstream makes successful innovation, if she is vertically separated, the welfare is

\[ W^M_{VS} = \left( \frac{\Delta}{2c_d + \Delta} \right)^2 (\Delta + 3c_d) \]

If she is vertically integrated, then only the integrated downstream firm makes investment, which is given by \( d_1 = \frac{\Delta}{c_d} \), and thus the social welfare is

\[ W^1_{VI} = \frac{\Delta^2}{2c_d} \]

Then we have

\[ W^M_{VS} - W^1_{VI} = \frac{\Delta^2}{4c_d(2c_d + \Delta)^2} (2c_d^2 - 2c_d\Delta - \Delta^2) \]

It is clear that \( 2c_d^2 - 2c_d\Delta - \Delta^2 \) is increasing in \( c_d \), and it is negative when \( c_d = \Delta \) and positive for \( c_d \) big enough. Therefore, there exists a \( \tilde{c}_d \) such that when \( c_d < \tilde{c}_d \), \( W^M_{VS} < W^1_{VI} \); and when \( c_d > \tilde{c}_d \), \( W^M_{VS} > W^1_{VI} \).

References


